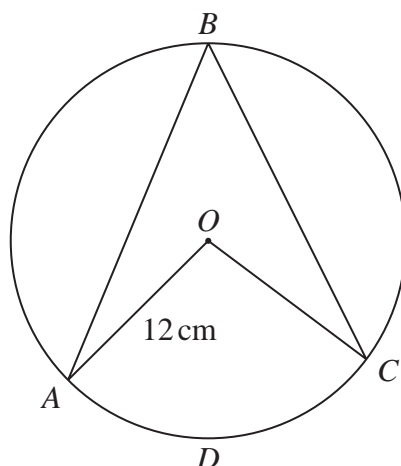**1**

Diagram **NOT** accurately drawn

A, B, C and D are points on a circle with centre O and radius 12 cm.

The area of the sector $OADC$ of the circle is 100 cm^2

Work out the size of angle ABC.
Give your answer correct to 3 significant figures.

Finding angle AOC :

Area of sector OADC = 100 cm^2

$$100 = \pi \times 12^2 \times \frac{\angle AOC}{360} \quad \text{①}$$

$$\text{angle AOC} = \frac{100}{\pi \times 12^2} \times 360$$

$$= \frac{250}{\pi} \quad \text{①}$$

Finding angle ABC:

$$\text{angle ABC} = \frac{1}{2} \times \text{angle AOC}$$

$$= \frac{1}{2} \times \frac{250}{\pi} \quad \text{①} = \frac{125}{\pi} = 39.8 \quad \text{①}$$

$$\underline{39.8} \qquad °$$

(Total for Question 1 is 4 marks)

2 *A*, *B*, *D* and *E* are points on a circle.
   ABC and *EDC* are straight lines.
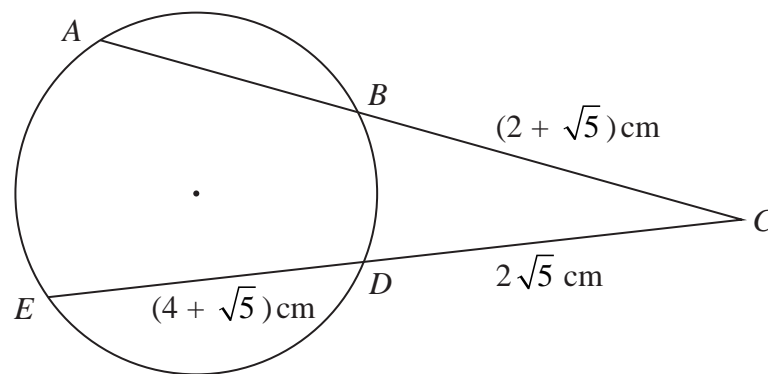

Diagram **NOT**
accurately drawn

$BC = (2 + \sqrt{5})\,$cm

$ED = (4 + \sqrt{5})\,$cm

$DC = 2\sqrt{5}\,$cm

Show that the length of *AB* is $(p\sqrt{5} + q)\,$cm, where *p* and *q* are integers whose values are
to be found.
Show your working clearly.

$$BC \times AC = CD \times EC$$

$$(2+\sqrt{5})(AB + 2 + \sqrt{5}) = 2\sqrt{5}(4 + \sqrt{5} + 2\sqrt{5}) \;①$$

$$AB = \frac{2\sqrt{5}(4 + \sqrt{5} + 2\sqrt{5})}{2+\sqrt{5}} - (2 + \sqrt{5}) \;①$$

$$= \frac{8\sqrt{5} + 2(5) + 4(5) - (4 + 4\sqrt{5} + 5)}{2+\sqrt{5}}$$

$$= \frac{8\sqrt{5} - 4\sqrt{5} + 10 + 20 - 9}{2+\sqrt{5}}$$

$$= \frac{21 + 4\sqrt{5}}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} \;①$$

$\downarrow$

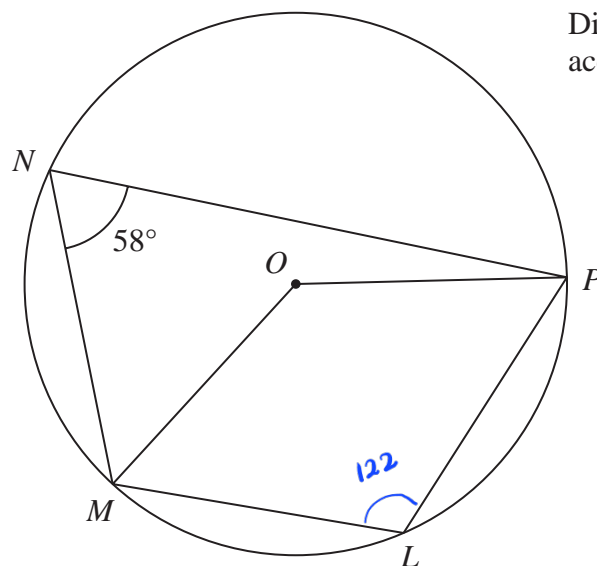$$= \frac{42 - 21\sqrt{5} + 8\sqrt{5} - 4(5)}{4 - 5} \quad \textcircled{1}$$

$$= \frac{22 - 13\sqrt{5}}{-1}$$

$$= -22 + 13\sqrt{5}$$

$$= 13\sqrt{5} - 22 \quad \textcircled{1} \quad \text{where} \quad p = 13$$
$$q = 22$$

(Total for Question 2 is 5 marks)

3

Diagram **NOT** accurately drawn

L, *M*, *N* and *P* are points on a circle, centre *O*

Angle *MNP* = 58°

(a) (i) Find the size of angle *MLP*

$180° - 58° = 122°$

122 ① °

(ii) Give a reason for your answer.

opposite angles in a cyclic quadrilateral sums up to $180°$. ②

(2)

(b) Find the size of the reflex angle *MOP*

reflex angle = $180° < \theta < 360°$

angle MOP = 2 × angle MLP

$= 2 × 122°$ ①

$= 244°$ ①

244 °

(2)

(Total for Question 3 is 4 marks)

4
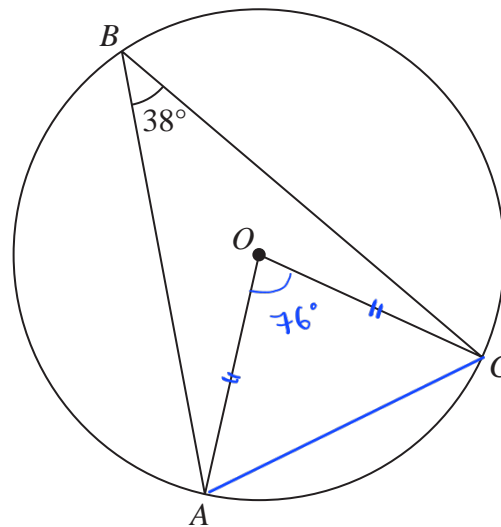
Diagram **NOT** accurately drawn

A, *B* and *C* are points on a circle, centre *O*.
Angle *ABC* = 38°

Work out the size of angle *OAC*.
Give a reason for each stage of your working.

$\angle AOC = 38 \times 2 = 76°$ ①
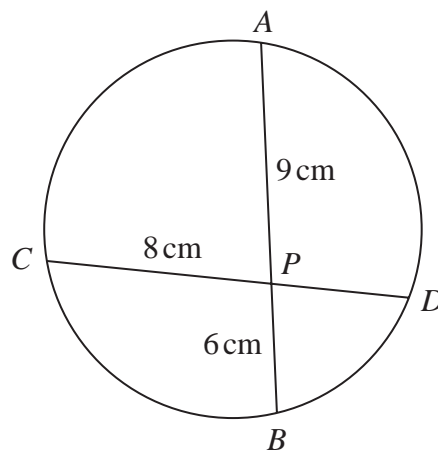
(Angle at centre is twice the angle at circumference) ①

$\angle OAC = \dfrac{180° - 76°}{2}$

$= 52°$

(Angle in a triangle adds up to 180°, base of isosceles are equal) ①

52 ① °

(Total for Question 4 is 4 marks)

5

Diagram **NOT** accurately drawn

APB and *CPD* are chords of a circle.

$AP = 9$ cm $PB = 6$ cm $CP = 8$ cm

Calculate the length of *PD*.

$$AP \times PB = CP \times PD$$

$$9 \times 6 = 8 \times PD \ \textcircled{1}$$

$$PD = \frac{9 \times 6}{8}$$

$$= 6.75 \ \textcircled{1}$$

6.75 .... cm
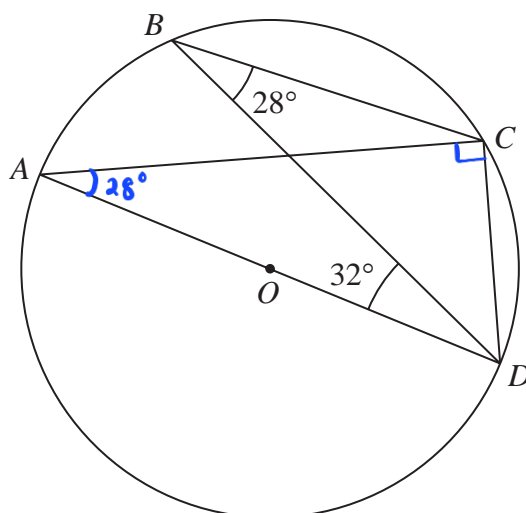
(Total for Question 5 is 2 marks)

6

Diagram **NOT**
accurately drawn

A, *B*, *C* and *D* are points on a circle, centre *O*.
AOD is a diameter of the circle.

Angle *CBD* = 28°
Angle *BDA* = 32°

Find the size of angle *BDC*.
Give a reason for each stage of your working.

- angle CAD = angle CBD = 28° ①

   (angle in the same segment are equal)

- angle ACD = 90° ①

   (angle in a semicircle is 90 degrees) ①

· angle BDC = 180° - 28° - 90° - 32°

            = 30° ①

   (angle in a triangle adds up to 180°)

30   ○

(Total for Question 6 is 4 marks)

7 *P*, *Q* and *R* are points on a circle, centre *O*.
TRV is the tangent to the circle at *R*.

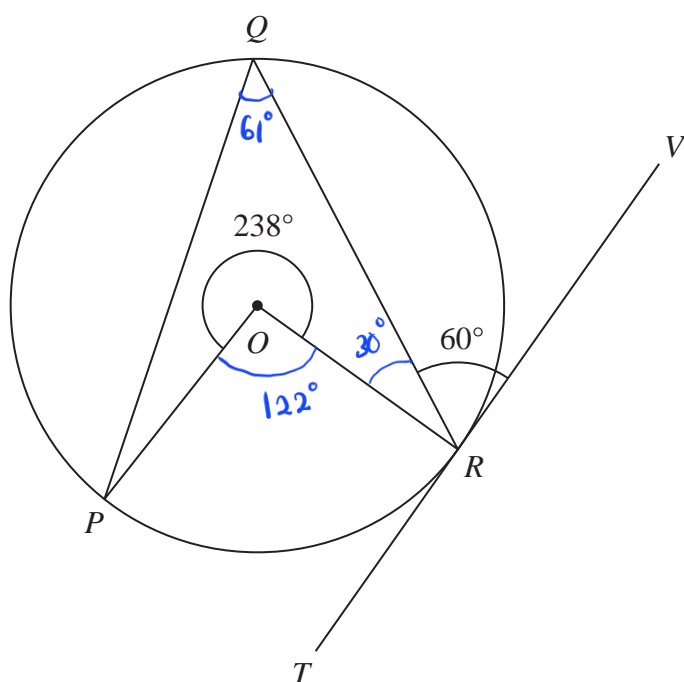Diagram **NOT** accurately drawn

Reflex angle *POR* = 238°
Angle *QRV* = 60°

Calculate the size of angle *OPQ*.
Give a reason for each stage of your working.

angle ORQ = 90° − 60°
$\qquad$ = 30°
(angle between a tangent and radius is 90°)

angle POR = 360° − 238°
$\qquad$ = 122° ①
(angle around a point is 360°)

angle PQR = $\frac{122°}{2}$ = 61° ①
(angle at centre of circle is twice the angle at circumference) ①

angle OPQ = 360° − 238° − 30° − 61°
$\qquad$ = 31° ①
(angle in quadrilateral = 360°)

$\dotfill$ 31 °

(Total for Question 7 is 4 marks)

8
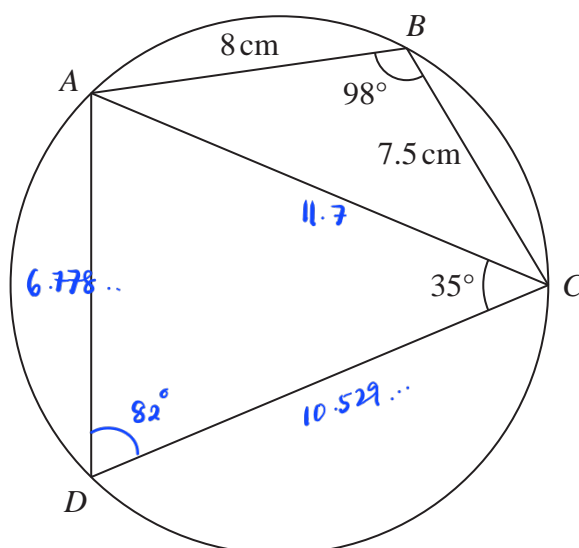
Diagram **NOT** accurately drawn

ABCD is a quadrilateral where *A*, *B*, *C* and *D* are points on a circle.

AB = 8 cm
BC = 7.5 cm
Angle *ABC* = 98°
Angle *ACD* = 35°

Work out the perimeter of quadrilateral *ABCD*.
Give your answer correct to one decimal place.

angle ADC = 180° − 98°

        = 82° ①

By using cosine rule :

$$AC^2 = 8^2 + 7.5^2 - 2(8)(7.5)\cos 98°$$

$$AC^2 = 136.95\cdots ①$$

$$AC = 11.702\cdots ①$$

By using sine rule :

$$\frac{AD}{\sin 35°} = \frac{11.702\cdots}{\sin 82°}$$

$$AD = \frac{11.702\cdots}{\sin 82°} \times \sin 35°$$

$$= 6.778\cdots ①$$

angle DAC $= 180° - 82° - 35°$

$\qquad = 63°$

By using sine rule :

$$\frac{DC}{\sin 63°} = \frac{6.778...}{\sin 35°}$$

$$DC = \frac{6.778....}{\sin 35°} \times \sin 63°$$

$$= 10.529..... \;①$$

Perimeter of ABCD $= 8 + 7.5 + 10.529... + 6.778....$

$\qquad = 32.8 \text{ cm} \;①$

32.8 .... cm

(Total for Question 8 is 6 marks)

9
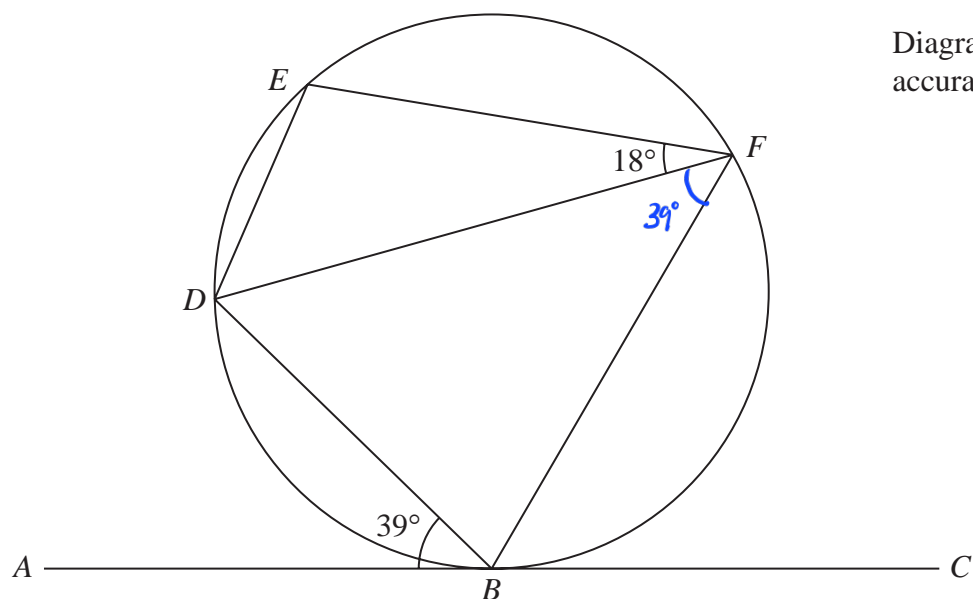
Diagram **NOT** accurately drawn

B, D, E and F are points on a circle.

ABC is the tangent at B to the circle.
Angle $ABD = 39°$
Angle $EFD = 18°$

Work out the size of angle BDE.
Give reasons for your working.

angle BFD = angle ABD = 39° ①
          (alternate segment theorem) ①

angle BDE = 180° − (18° + 39°) ①
      = 180° − 57°
      = 123° ①
(opposite angles in a cyclic quadrilateral sum up to 180°)

123 °

(**Total for Question 9 is 4 marks**)

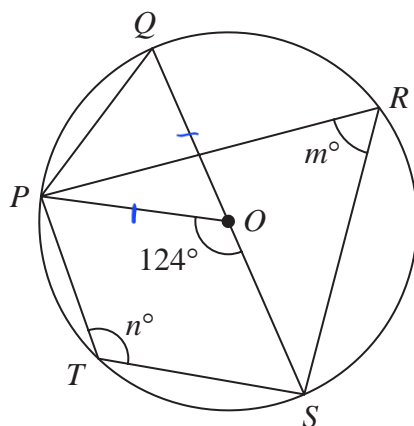**10** *P, Q, R, S* and *T* are points on a circle with centre *O*.

Diagram **NOT**
accurately drawn

QOS is a diameter of the circle.

angle *POS* = 124°   angle *PRS* = *m*°   angle *PTS* = *n*°

(a) Find the value of

   (i)   *m*

     124° ÷ 2 = 62°

                                    62°

   (ii) *n*

     180° − 62° = 118°

                                    118°

                                    (2)

(b) Find the size of angle *QPO*.

   PQO = 62°

   So, *QPO* must be 62° as the triangle is isosceles

                                    62 °

                                      (1)

                         (Total for Question 10 is 3 marks)
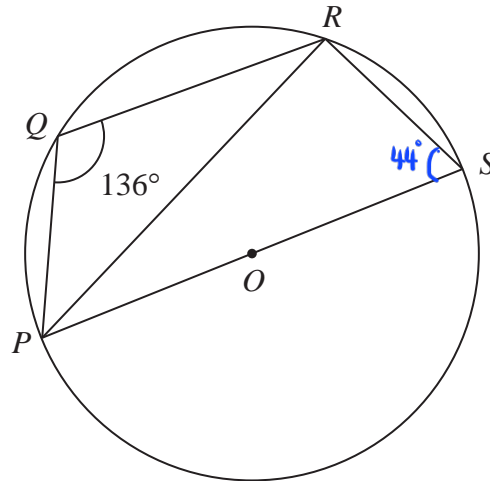
11

Diagram **NOT** accurately drawn

P, *Q*, *R* and *S* are points on a circle with centre *O*

PS is a diameter of the circle.

Angle *PQR* = 136°

Work out the size of angle *RPS*

angle PSR : 180° - 136°

        : 44° (Opposite angles in a cyclic quadrilateral sums up to 180°)

angle PRS = 90° (angle inside a semicircle is always 90°)

angle RPS = 180° - 44° - 90°

        = 46° (angles in a triangle add up to 180°)

46 °

(Total for Question 11 is 3 marks)

12
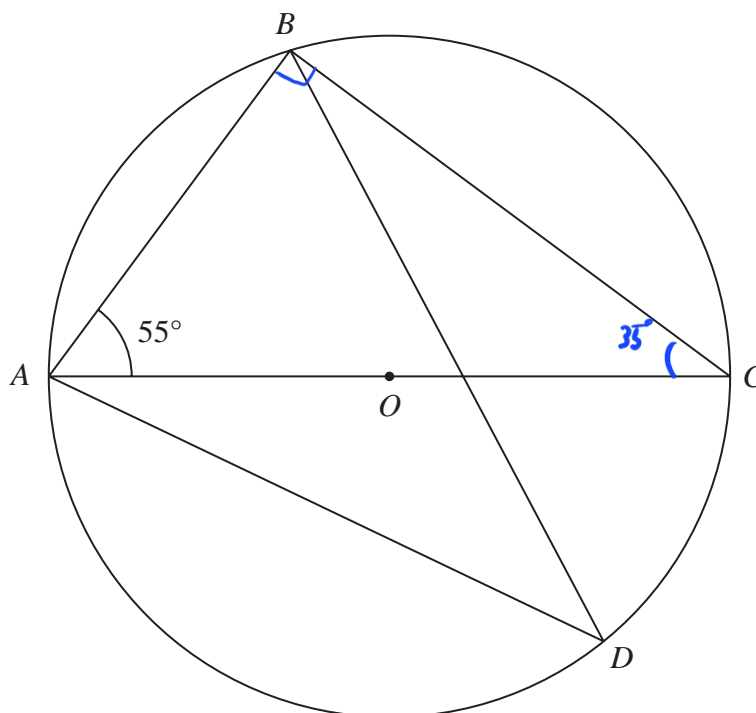
Diagram **NOT** accurately drawn

A, *B*, *C* and *D* are points on a circle, centre *O*
AOC is a diameter of the circle.

Angle *BAC* = 55°

Work out the size of angle *ADB*
Give a reason for each stage of your working.

$ABC = 90°$ ①
(angles in semicircle are 90°)

$ACB = 180° - 90° - 55°$
$\quad = 35°$ ①
(angles in a triangle add up to 180°)

①

$ADB = ACB = 35°$
(angles in the same segment are equal)

35 ○

(Total for Question 12 is 4 marks)

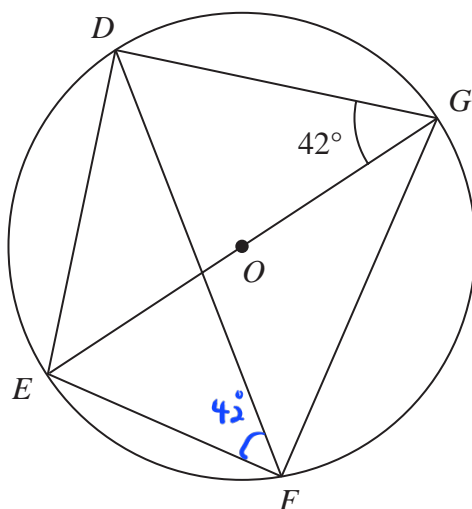**13** *D*, *E*, *F* and *G* are points on a circle, centre *O*

Diagram **NOT** accurately drawn

EOG is a diameter of the circle.

Angle *EGD* = 42°

Calculate the size of angle *DFG*
Give a reason for each stage of your working.

DFE = 42° ①

(angles in same segment are the same) ①

EFG = 90°
(angles in a semicircle are 90°) ①

DFG = 90° - 42°

= 48° ①

Angle *DFG* = ...48........ °

(Total for Question 13 is 4 marks)

14 *A*, *B*, *C* and *D* are points on a circle, centre *O*
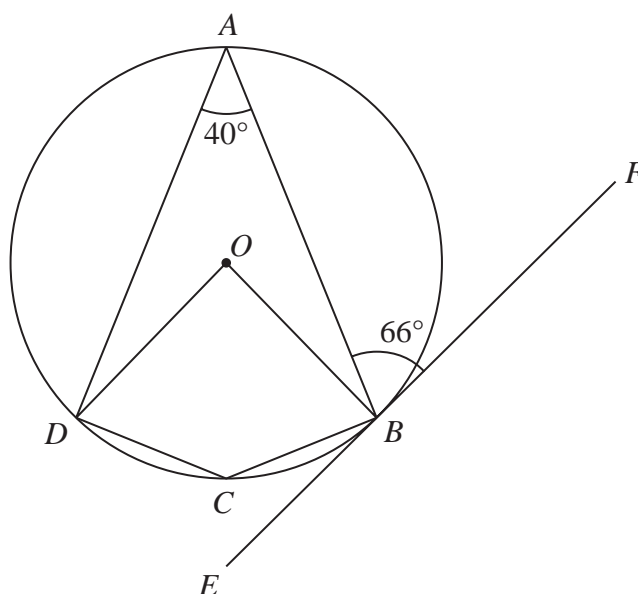
EBF is the tangent to the circle at *B*

Diagram **NOT** accurately drawn

(a) (i) Work out the size of angle *DCB*

140 ° (1)

(ii) Give a reason for your answer to (a)(i)

Opposite angles in cyclic quadrilateral add up to 180° (1)

(b) Work out the size of angle *ADO*

ABO = 90 - 66 = 24

DOB (reflex) = 140 × 2 = 280

ADO = 360 - 40 - 280 - 24

= 16

16 °

(3)

(Total for Question 14 is 5 marks)

15 *A*, *B* and *C* are points on a circle.
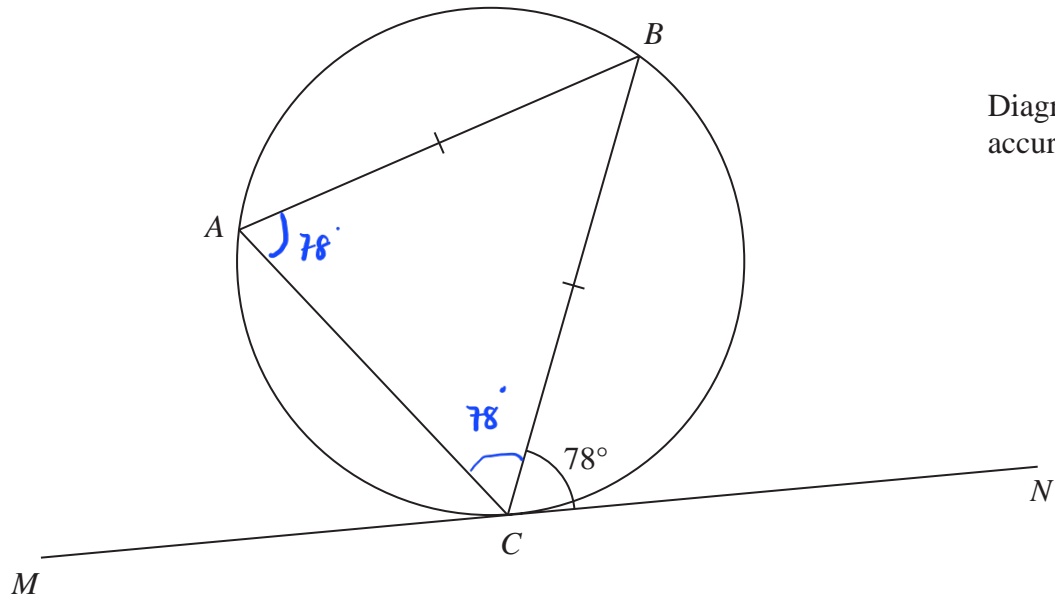
Diagram **NOT** accurately drawn

MN is the tangent to the circle at *C*

AB = CB

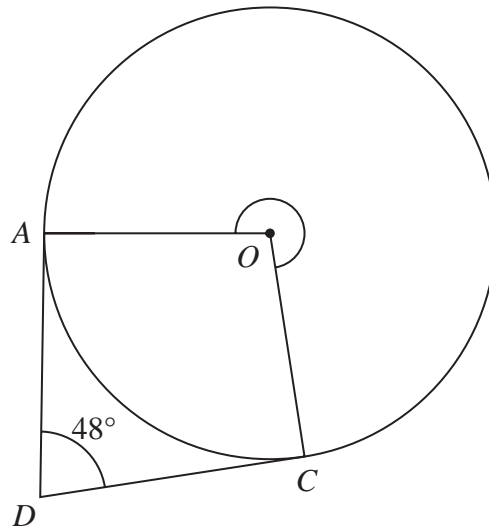Angle *BCN* = 78°

Find the size of angle *ABC*

angle BAC = angle BCA = 78°

angle ABC = 180 − 78 − 78 ①

= 24 ①

24 °

(Total for Question 15 is 2 marks)

16

Diagram **NOT**
accurately drawn

A and *C* are points on a circle, centre *O*

DA is the tangent to the circle at *A* and *DC* is the tangent to the circle at *C*

Angle *ADC* = 48°

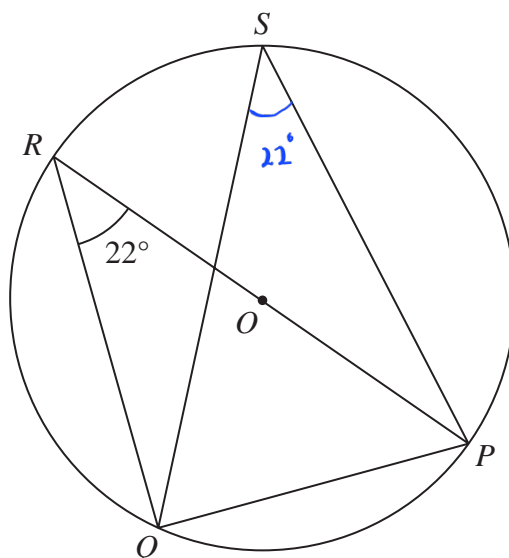Work out the size of reflex angle *AOC*

$$DCO = DAO = 90° \; ①$$

$$AOC = 360° - 48° - 90° - 90°$$

$$= 132° \; ①$$

$$AOC \; (reflex) = 360° - 132°$$

$$= 228° \; ①$$

228 °

....

(Total for Question 16 is 3 marks)

17

Diagram **NOT** accurately drawn

P, *Q*, *R* and *S* are points on a circle, centre *O*
ROP is a diameter of the circle.
Angle *PRQ* = 22°

(a) (i) Find the size of angle *RQP*

90 °

(1)

(ii) Give a reason for your answer.

angle in a semicircle is 90°

(1)

(b) (i) Find the size of angle *PSQ*

22 °

(1)

(ii) Give a reason for your answer.

Angles in the same segment are equal.

(1)

(Total for Question 17 is 4 marks)